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Haarwavelet and Its Application for Problem Solving In Optimal Control System

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Abstract-The aim of this project is to analyze and analytical solution of different optimal control problems are difficult to obtain because they involve differential equations with single or multiple boundary conditions. In this dissertation, numerical methods using Haar Wavelet a represented to overcome this difficulty. The method reduces the differential equations into a set of line a matrix algebraic equation. The nice properties of Haar Wavelet like compact support in time and multi resolution are shown to reduce the computational complexity to a great extent. The presented method is applied to achieve the optimal control for time varying and time invariant performance indices.
Key words: Haar wavelet, Short-time Fourier transform, optimization theory, variation problem.

I. INTRODUCTION

Haar wavelet transform for solving optimal control problem shave evolved significantly over the past thirty- forty years again maximum principle. Most early methods were base don't finding a solution that satisfied the maximum principle, or related necessary conditions, rather than attempting a direct minimization of the objective function(subject to constraints) of the optimal control problem. For this reason, methods using this approach are called indirect methods[1-5].

The main draw back find indirect methods is their extreme lack of robustness: the iterations of an indirect method must start close, sometimes very close, to a local solution in order to solve the two-point boundary value sub problems. Additionally, since first order optimality conditions are satisfied by maximizes and saddle points as well as minimizers, there is no reason, in general, to expect solutions obtained by indirect methods to be minimizers.

Both of these drawbacks of indirect methods are overcome by so-called direct methods. Direct methods obtain solutions through the direct minimization of the objective function (subject to constraints) of the optimal control problem. In this way the optimal control problem is treated as an infinite dimensional mathematical programming problem. Thus, the computational advantages and drawbacks of indirect methods are fully determined by the properties of the respective boundary value problem. In direct methods, the extreme of the functional is found directly.

The wavelet applications in dealing with dynamic system problems, especially in solving partial differential equations with two-point boundary value constraints have been discussed in many papers[9-12]. By transforming differential equations into algebraic equations, the solution may be found by determining the corresponding coefficients that satisfy the algebraic equations.

There is a new direct computational method to solve variation problems[13] via Haar transform by taking advantage of the nice properties Therefore, the differential and integral expressions which arise for the system dynamics and the performance index, the initial or boundary conditions, or even for general multi point boundary conditions are converted into algebraic equations with unknown coefficients. In this way, the optimal control problem is replaced by a parameter optimization problem, which consists of the minimization or maximization of the performance index, subject to algebraic constraint.

In this period several methods for the study of optimal control problems have been developed of which the maximum principle and dynamic programming play the most important role. These two general methods are attractive mathematical means form treating optimal control problems but they have some limitations which considerably decrease their computational efficiency. **2004**, Chun[13] establishes a clear procedure for the variation problem solution via Haar wavelet technique. The variation problems are solved by means of the direct method using the Haar wavelets and reduced to the solution of algebraic equations. A Haar wavelet-based method for optimal control of the second-order linear systems with respect to a quadratic cost function for any length of time is proposed. A Haar wavelet integral operational matrix and properties of the product are used in finding the approximate solutions of optimal trajectories and optimal control by solving only two algebraic equations instead of solving the Riccati differential equation[23]. **2006**, worked further on their proposed Haar wavelet-based control of time-varying state-delayed systems with a computational method. The above work has given the direction of further investigate for improvement in the performance of Direct method for solving variation problem. It has become a motivation to work.

II. WAVELET THEORY

Wavelet transforms or wavelet analysis is a recently developed mathematical tool for many problems. Wavelet techniques unable us to cut complicated function into several simpler ones and study them separately. This property along with fast wavelet algorithms makes these techniques very attractive in analysis and synthesis problems. Different types of wavelet have been used as a tool to solve problems, in signal analysis, image analysis, medical diagnostics, boundary value problems, geophysical signals processing, statistical analysis, pattern recognition and many others. Wavelets also can be applied in numerical analysis.While wavelet has gained popularity in these areas, new application is continually being investigated. A reason for the popularity of wavelet is its effectiveness in representation of non-stationary signal (transient). Since most of natural and human made signals are transient in nature, different wavelets have been used to represent this much larger class of signals than Fourier representation of stationary signals. Unlike Fourier based analysis that use global (non local) sine and cosine functions as a bases, wavelet analysis uses bases that localized in time and frequency to represent non stationary signal more effectively.

The use of wavelets in signal processing applications is continually increasing. This use is partly due to the ability of wavelet transforms to present a time-frequency (or timescale) representation of signals that is better than that offered by Short-time Fourier transform (STFT). Unlike the STFT, the wavelet transform uses a variable-width window (wide at low frequencies and narrow at high frequencies) which enables it to "zoom in" on very short duration high frequency phenomena like transients in signals.

Wavelet representation is much more compact and easier to implement using the powerful multi resolution analysis. One can represent a signal by finite some of component at different resolution so that each component can processed adaptively based on the objectives of application. This capability to represent signal compactly & inseveral level of resolution is the major strength of wavelet analysis.

III. TYPES OF WAVELETS

Wavelet transforms comprise an infinite set [16]. The different wavelet families make different trade-off between how compactly the basic functions are localized in space and how smooth they are functions that satisfy the requirements of time and frequency localization. The necessary and sufficient condition for wavelets is: it must be oscillatory, must decay quickly to zero, and must have an average value of zero. In addition, for the discrete wavelet transform considered here, the wavelets are orthogonal to each other. Fourier basis functions are localized in frequency but not in time while wavelets are localized in both time (through translation) and frequency (through dilation). Wavelets can provide multiple resolutions in both time and frequency unlike STFT. As compared to the traditional Fourier analysis, the signal can be accurately reproduced with the wavelet analysis using a relatively small number of components.

Moreover, many classes of functions can be represented by wavelets in a more compact way. The analyzing wavelets are called the "mother wavelets" and the dilated and translated versions are called the "daughter wavelets." The "mother wavelet" determines the shape of the components of the decomposed signals. There are many types of wavelets such as Haar (H), Daubechies 4 (D4), Daubechies 8 (D8), Coiflet 3 (C3) Symmlet 8 (S8), and so on. A particular type of wavelet is selected depending on the particular type of application. Similar to Fourier transform, it has a digitally implement able counterpart, the discrete wavelet transform (DWT). The generated waveforms are analyzed with wavelet MRA to extract sub-band information from the simulated transients. Daubechies eight (D8) wavelet is localized (i.e., compactly supported in time and, hence, is good for short and fast transient analysis).

Wavelet coefficients of the signal are derived using matrix equations based on decomposition and reconstruction of a discrete signal using Mallat's algorithm.

Unlike Fourier analysis, which relies on a single basis function, wavelet analysis uses basis functions of a rather wide functional form. One often hears of these different wavelets named after their inventors (e.g.Daubechies wavelets (named for ingridDebauchies, a lead researcher in wavelet theory),the Morlet wavelet, Meyer wavelets, and Coiflet wavelets).some of these functions are shown in figure 2.1.

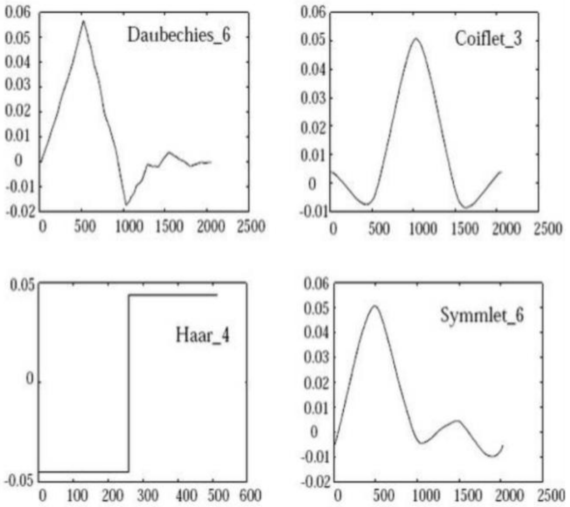


Fig 1 Different types of wavelet function

IV. ADVANTAGE

The advantage of wavelet analysis over Fourier analysis may not be obvious to the casual observer. This is the case because Fourier methods have been the standard tool in many kinds of signal analysis. However the Fourier series and Fourier transform are not without their limitation. For instance, a Fourier series requires that all time functions involved be periodic. Fourier transform have wide bandwidth for short term transients. One may argue that it would take an infinite amount of time using both past and future data to extract the spectral information at a single frequency. In addition, Fourier analysisdoes not consider frequencies that evolve in time. Finally, Fourier techniques suffer from certain annoying anomalies such as Gibb' phenomenon and aliasing (with the fast Fourier transform).

The frequency of Fourier analysis in dealing with transient is often circumvented by windowing the input signal so that the sampled values will converge to zero at the end points.
These short-time Fourier transforms (STFT) and Gabor transforms have been under development since the mid- 1940s, but have met with mixed success. The main disadvantage to windowing is that the window is fixed, and, as frequency is increased, there are more and more cycles within the window. Thus, individual frequency components are not treated in the same way.

Wavelets, on the other hand, can be chosen with very desirable frequency and time characteristics as compared to Fourier techniques. The basic difference is as follows: in contrast to the STFT, which uses a single analysis window, the wavelet transform uses short windowsat high frequencies and long windows at low frequencies. Thus thewindowing of wavelet transforms is adjusted automatically for low or high frequencies.

A. Fourier Transform and its Limitations

The one-dimensional Fourier Transform (FT) and its inverse are specified. In the view of the above definitions, FT can be though of as just a generalized version of the Fourier series the summation is replaced by the integral and a 'countable' set of basis functions is replaced by a Uncountable one this time consisting of cosines for the real part and sines for the complex part of the analyzed signal(which can now be a complex number, that is another generalization of FT).

Due to the fact that each of the basis functions spans over the total length of the analyzed signal, this creates what is known as time-frequency resolution problem, which is best illustrated with an example.

Suppose the Fourier transform of a simple signal containing a superposition of four different frequencies (namely 5, 10, 20 and 50Hz) is taken$x(t)$.

B. Time domain signal

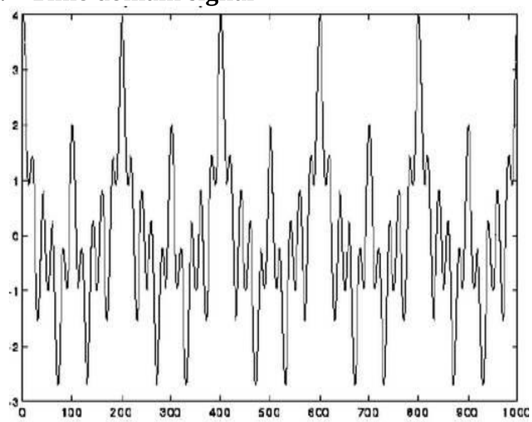


Fig 2.Sampled cosine signal with 5, 10, 20,50Hz at all times.

C. Frequency Domain Signal

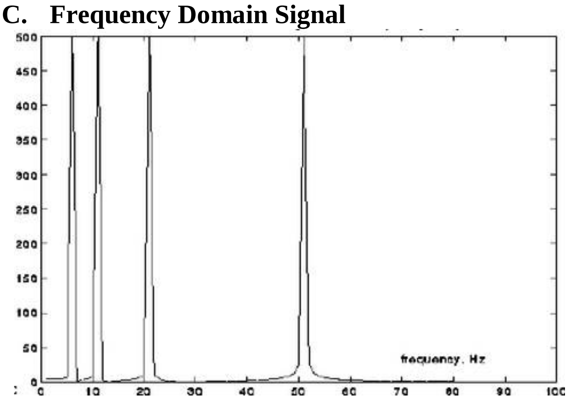


Fig 3. Frequency transform of the cosine signal with 4 frequency components

The Fourier spectrum clearly reveals the presence of these four frequencies. Now consider a signal contains the same frequencies as the previous one, only this time they are not superimposed, but appear sequentially as the picture below shows.

D. Different Signal

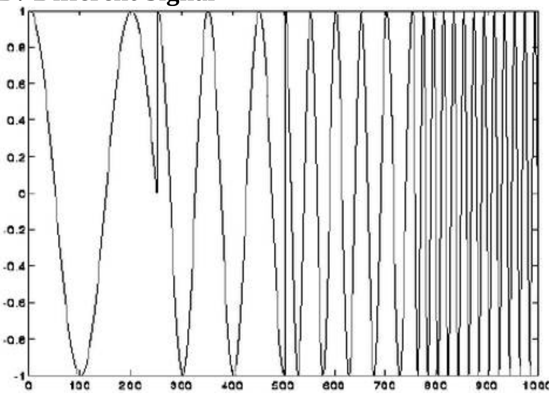


Fig 4. Same frequencies at different times

E. Virtually the same Fourier spectrum

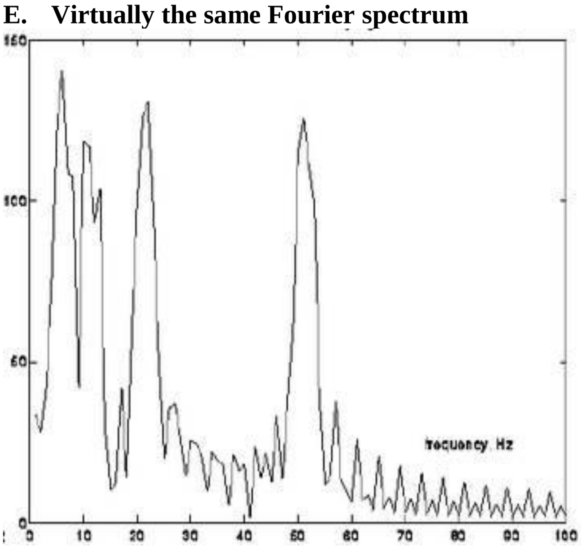
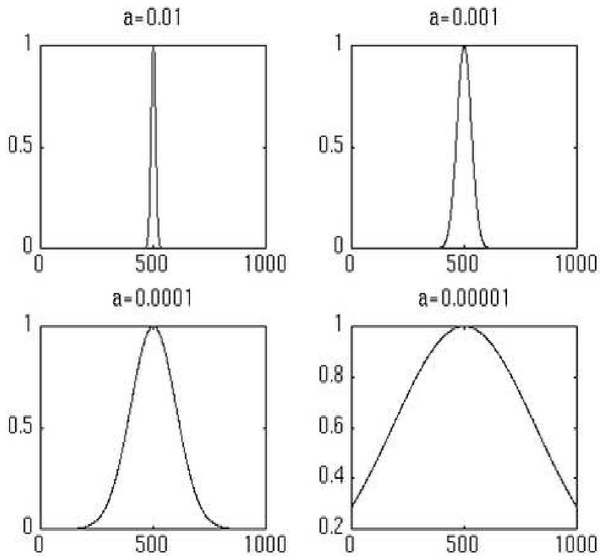
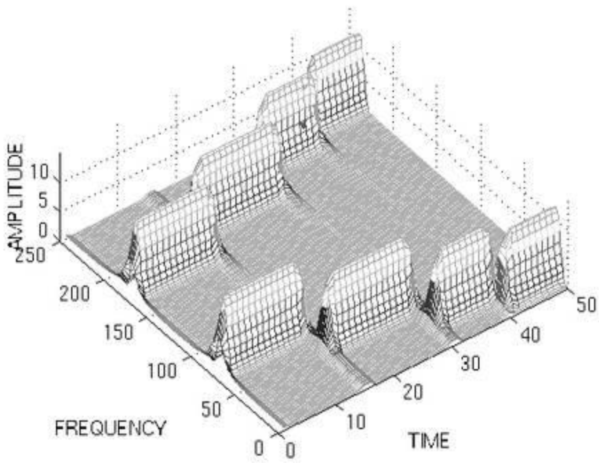


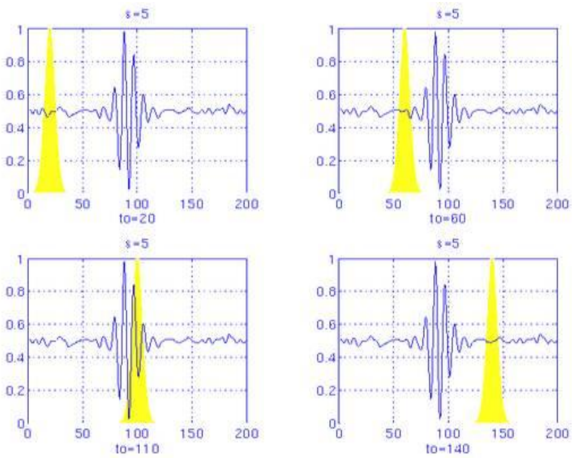
Fig 5. FT of the non stationary signal

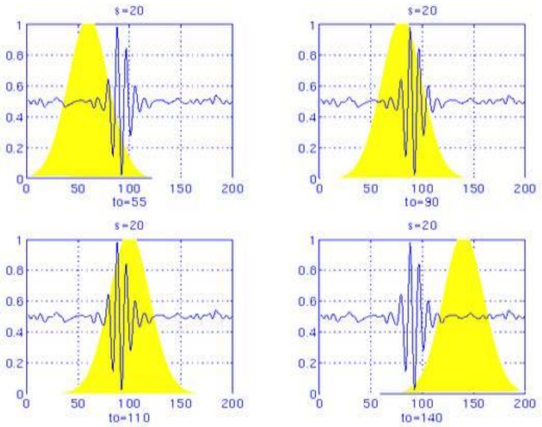
It should be noted how similar the frequency-domain representations of both signals are. (The small peaks are purely a consequence of discontinuities present in the second signal ("ringing") and could be completely eliminated if a signal was chosen more carefully.)

The important conclusion is that FT does give information about what frequencies are present in the signal; however it lacks the ability to correlate the frequencies with the time of their presence. This fact does not even come into consideration when only stationary signals are processed; however as soon as non- stationary signals are to be analyzed, it becomes a serious handicap.









V. VARIATIONAL PROBLEM

Numerical methods of variational calculus are usually subdivided into two major classes: indirect and direct methods. Indirect methods are based on the use of necessary optimality conditions (Variational calculus; Euler equation; Weierstrass conditions (for a variationalextremum); Transversality condition; Pontryagin maximum principle), with the aid of which the original variational problem is reduced to a boundary value problem. The subdivision of the numerical methods of variational calculus into direct and indirect methods is largely arbitrary. In some algorithms both approaches are utilized. Moreover, some methods cannot be classified as belonging to either class. Thus, methods based on sufficient optimality conditions form a separate group.

In the large number of problems arising in analysis, mechanics, geometry, and so forth, it is necessary to determine the maximal and minimal of a certain functional. Because of the important role of this subject in science and engineering, considerable attention has been received on this kind of problems. Such problems are called variational problem. The branch of numerical mathematics which deals with the determination of extremal values of functional is called Variational calculus

VI. APPLICATION

In seismic and geological signal processing as well as medical and biomedical signal and image processing, wavelet transforms are used for denoising, compression, and detection. In general, there are many examples of successful application of wavelets in signal and image processing: speech coding, communications, radar, sonar, denoising, edge detection, and feature detection. Wavelet transform is also used in multi-scale models of stochastic processes and analysis. Orthogonal wavelets are very successful in numerical analysis like solving partial differential equations, speech coding and other similar applications, where symmetry is not a major requirement. Daubechies wavelets are very good in terms of their compact representation of signal details. They are,however, not efficient in representation of signal approximation at a given resolution.

Wavelet transform has found many applications in applied mathematics and signal processing. Due to its zooming property, which allows a very good representation of discontinuities, wavelet transform is successfully used in solving partial differential equations. They give a generalization of finite element method and, due to their localizing ability, provide sparse operators and good numerical stability.

VII. CONCLUSION

The method of using the Haar wavelets to solve variational problems reduces variational problems to the solution of algebraic equations, and so the calculation is straight forward and digital computer oriented. Some fundamental properties on Haar wavelets have been derived, and some effective algorithms have been applied to solve the rather difficultvariational problems successfully. So after theoretical reasoning and numerical demonstrations, It can be concluded as follows

The Haar wavelets approach is a powerful tool for numerical analysis. The integration matrix P for Haar wavelets provides an important connection between the Haar wavelets transform and the dynamic systems analysis. Optimisation problems with various constraints have been solved algebraically and operationally via Haar wavelets. The main advantages of this method is its simplicity and small computation costs, it is due to the sparcity of the transform matrices and to the small number of significant wavelet coefficients.

VIII. FUTURE SCOPE

Direct methods produce less accurate solutions than indirect methods. By solving numerically several difficult optimal control problems from aeronautics, it is found that in practice the minimum functional value is obtained with relative low accuracy (i.e.errors of about one percent). does not necessarily yield better values for the extremely complicated problems arising from aerodynamics. However, this "quantity" of one percent can be a crucial part of the payload in a space flight mission.

A second disadvantage is that the discretized optimal control problems have sometimes several minima. Applying the direct methods to the discretized problem they often end up in one of these "pseudominima". This solution, however, can be quite a step away from the true solution satisfying all

the necessary conditions from variational calculus resulting.
To overcome the observed disadvantages of the direct method it would be desirable to combine the good convergence properties of the direct method with the reliability and accuracy of the multiple shooting methods.

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